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New Mixing-Length Model for Supersonic Shear Layers

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Introduction

EARLY experiments on supersonic free shear layers revealed the inverse relationship between spreading rate and increasing Mach number.[1] It was thought that the reduction in spreading rate at high Mach number was due to a density effect. The experiments by Brown and Roshko[2] had shown that there is some effect of density ratio on spreading rate; however, the effect is much smaller than that observed in supersonic shear layers.

In an attempt to correlate the various data on high-speed shear layers, Bogdanoff[3] introduced the concept of the convective Mach number, which in effect casts the data into a frame convecting at the velocity of the large eddies of the mixing layer. Many investigators had attempted to incorporate compressibility effects in turbulence models, but their attempts were essentially a curve fit of spreading rate vs Mach number data, and a subsequent damping of the eddy viscosity.

A new mixing-length model has been developed without empirically fitting the data. A rationale is presented for prescribing the mixing length which reduces to the well-established incompressible form for subsonic flows, and produces the correct spreading behavior as the Mach number is increased above unity.

Theory

From Prandtl's mixing-length theory, the turbulent viscosity can be written as follows[4]:

$$\mu_t = \rho \ell^2 \left| \frac{\partial u}{\partial y} \right| = \rho C^2 L^2 \left| \frac{\partial u}{\partial y} \right| \tag{1}$$

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where ℓ is the mixing length. The mixing length is generally the product of an empirically determined constant C and some characteristic scale L of the mixing region. For incompressible shear-layer calculations, the shear-layer width δ serves as the characteristic length and the value of the constant ranges from 0.05 to 0.09. When the above formula is used for calculation of free shear layers at high Mach number, it fails to predict the reduced spreading rate. It is thus felt that compressibility effects should be included in the mixing-length definition. The main difference between incompressible and supersonic shear layers is that, at some regions in supersonic shear layers, relative supersonic flow can exist. A relative supersonic flow exists at a given axial location when there is a flow which moves supersonic relative to the other flows within the shear layer. The mixing length for a turbulent shear layer can be interpreted analogous to the mean free path of a molecule in the derivation of the laminar viscosity from kinetic theory. The mixing length can then be assumed proportional to some distance over which a disturbance can propagate or penetrate.

In subsonic shear layers there is no relative supersonic flow, therefore the characteristic scale L can be assumed to be δ. For supersonic shear layers, L can be assumed to be the distance between the points where flow moves sonic relative to the local point, at a given streamwise station. Figure 1 shows a velocity profile for a supersonic shear layer, At a given point y, where the velocity is u and the speed of sound is a, one can find the locations, Y_a and Y_b, where the flow is moving sonic relative to the point y with a limitation that Y_a and Y_b cannot exceed the edges of the shear layer, Y_{el} and Y_{eu}. If a relative sonic flow exists, the values of $|(u_a - u)/a|$ and $|(u_b - u)/a|$ are unity and L is $|Y_a(y) - Y_b(y)|$. Using the above assumption, the turbulent viscosity for supersonic shear layers can be written as follows:

$$\mu_t = \rho C^2 C_s^2 L^2(y) \left| \frac{\partial u}{\partial y} \right| \tag{2}$$

where $L(y)$ is the new characteristic scale obtained as described and C_s is a constant which can be determined from experimental data. By matching the calculated spreading rate parameter for a Mach 5 free shear layer to 37, which was obtained experimentally, the value of 0.89 was obtained for C_s. If there is no relative supersonic flow within the shear layer, C_s becomes unity.

Results and Discussion

To verify the new mixing-length model, a two-stream air mixing layer with the following conditions was calculated:

p is constant across shear layer and $T_{t1} = T_{t2}$
$P_r = 0.72$, $P_{rt} = 0.9$, and $T_1 = 300°$ K
$M_1 = 0.1$, 1, 2, 3, 4, 5 and $Re\,1/m = 2.2 \times 10^7$
$u_2/u_1 = 0.005$, $C = 0.085$ and $C_s = 0.89$

The boundary-layer code STAN5[5] with the following two definitions of L were used for the calculations:

$$L_a = Y_{0.99} - Y_{0.01} \tag{3}$$

$$L_b = L(y) \text{ from Fig. 1} \tag{4}$$

Y_s is the y coordinate where $(u - u_2)/(u_1 - u_2) = s$ and Y_{eu} and Y_{el} are the edges of the shear layer, $Y_{0.99}$ and $Y_{0.01}$, respectively.

L_a is from the incompressible mixing-length model in Eq. (1) and L_b is from the new mixing-length model in Eq. (2). To calculate the spreading-rate parameter, the following formula was used:

$$\sigma = F \times 1.855 \times (X_a - X_b)/(DY_a - DY_b) \tag{5}$$

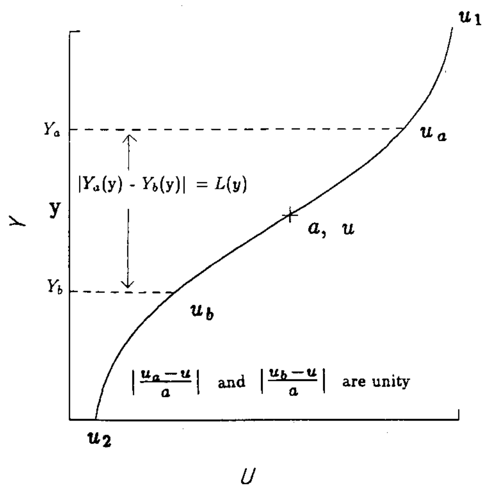


Fig. 1 Explanation of the new mixing-length concept.

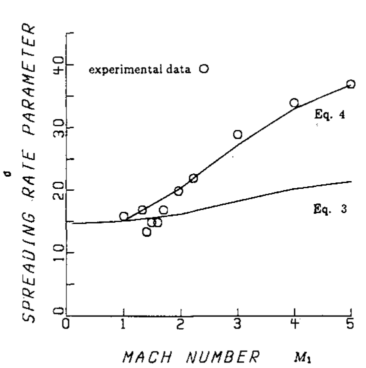
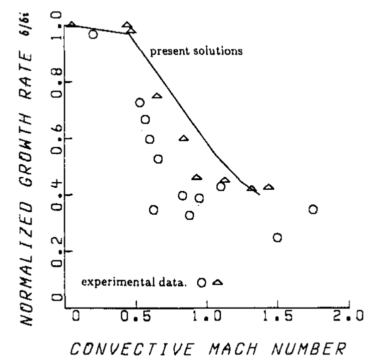


Fig. 2 Variation of spreading-rate parameter σ with Mach number.



$$M_{c1} = M_1 \frac{(1 - u_2/u_1)}{(1 + 1/\sqrt{\rho_2/\rho_1})}$$

Fig. 3 Variation of normalized growth rate δ/δi with convective Mach number.

where X_a and X_b are axial coordinates and DY_a and DY_b are the distances between $Y_{0.9}$ and $Y_{0.1}$ at X_a and X_b, respectively. $F(=0.905)$ is a correction factor that converts the spreading rate for $u_2/u_1 = 0.05$ to that for $u_2/u_1 = 0$.

Figure 2 shows the spreading-rate parameter variations with Mach number from calculations and measurements. As seen from this figure, the new mixing-length model predicts a significant decrease of the spreading rate with increasing Mach number and the results agree well with data, while the incompressible mixing-length model gives a very small decrease of spreading rate. Figure 3 plots the variation of the normalized spreading rate with convective Mach number, and includes the experimental data compiled in Refs. 3 and 4.

The spreading-rate behavior from the new mixing-length definition shows the same trend as the experimental data. The spreading rate from the present solutions start to rapidly decrease when the convective Mach number M_{c1} is greater than 0.46 ($M_1 = 1$), because then a relative sonic flow exists in the shear layer.

Conclusion

A new mixing-length model was developed and applied to include the compressibility effect. The characteristic scale of the mixing region is determined locally by the lateral distance between the two points where flow moves sonic relative to the local point. Supersonic free shear layers at various Mach numbers were calculated with the new model. The results show the decrease of spreading rate with increasing Mach number and agree well with the existing experimental data.

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Improvements to a Nonequilibrium Algebraic Turbulence Model

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Introduction

At the Viscous Transonic Airfoil Workshop held at the AIAA 25th Aerospace Sciences Meeting at Reno, Nevada, in January 1987, Coakley[1] and King[2] both showed that the nonequilibrium turbulence model of Johnson and King[3,4] performed significantly better than the more widely used equilibrium models of Cebeci and Smith[5] and Baldwin and Lomax[6] for separated transonic airfoil flows. However, for some easier test cases where the shock wave on the upper airfoil surface was too weak to cause separation (RAE 2822 airfoil test cases 6 and 9 of Cook et al.[7]), the shock location was better predicted by the aforementioned equilibrium models.

An examination of numerical solutions for the RAE 2822 airfoil has revealed that most of the observed differences in

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